TRINITY COLLEGE

Mathematics Methods Unit 3,4
Test 4 2016

Section 1 Calculator Free

Calculus Involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME _____

DATE: Friday 22 July **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine the equation of the tangent to the curve $y = x\ln x$ at the point (e,e)

2. (4 marks)

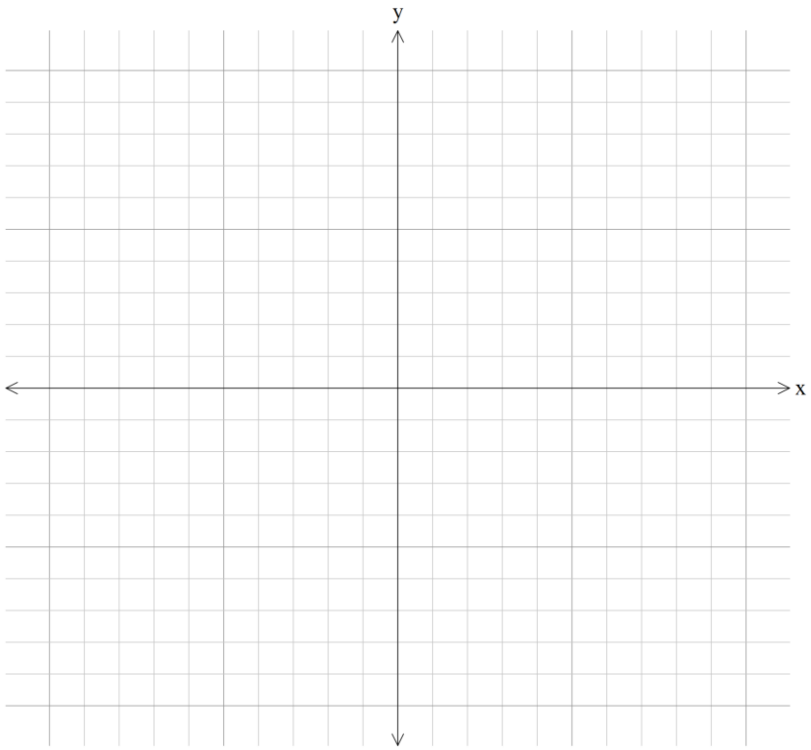
(a) $\int \frac{\sin x}{1+\cos x}\,dx$ [2]

(b) $\int \frac{8-6x^2}{x^3-4x+1}\,dx$ [2]

3. (5 marks)

Consider the functions $y = 3 - x$ and $xy = 2$.

(a) Draw a sketch of these two functions which clearly shows the enclosed area. [2]



(b) Determine the exact value of the enclosed area. [3]

4. (12 marks)

Differentiate each of the following functions. Do NOT simplify.

(a) $y = \ln \dfrac{2x}{x^2 - 1}$ [3]

(b) $y = \ln \tan 2x$ [3]

(c) $y = \ln\ln x^2$ [3]

(d) $y = \ln(e^x(1 - e^{-x}))$ [3]

TRINITY COLLEGE

Mathematics Methods Unit 3,4
Test 4 2016

Section 2 Calculator Assumed

Calculus Involving Logarithmic Functions, Continuous Random Variables

STUDENT'S NAME _____

DATE: Friday 22 July **TIME:** 30 minutes **MARKS**: 29

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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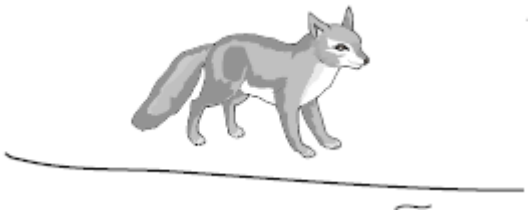
5. (5 marks)

The time, *t*, in hours that a fox spends hunting each night is a continuous random variable with probability density function

$$f(t) = \begin{cases} \frac{k}{32}t(4-t) & \text{for } 0 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$



(a) Determine the value of k. [3]

(b) Calculate the probability the fox spends more than 3 hours hunting on one night. [2]

6. (18 marks)

The time, in minutes, between telephone calls received at a pizza shop is a continuous random variable, T , with a density function given by

$$f(t) = \begin{cases} 0.25e^{-0.25t} & \text{for } t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Calculate the probability that the next call occurs within 8 minutes. [2]

(b) Calculate the probability that the next call occurs between 3 and 6 minutes given it occurs within 8 minutes. [2]

(c) Determine the expected time to the next call. [3]

(d) Determine the interval of time that is within one standard deviation of the expected completion time. [4]

(e) Determine $\text{Var}(1-2T)$, where Var is the variance. [2]

(f) (i) For the random variable T, give the cumulative distribution function $F(t)$. [3]

(ii) Determine $P(T \geq 10)$ [2]

7. (6 marks)

X is a continuous random variable, denoting the number of minutes in excess of two hours which a person takes to travel from one town to another. The probability density function is defined as follows.

$$f(x) = \begin{cases} k(10+x) & -10 \le x < 0 \\ k(10-x) & 0 \le x \le 10 \\ 0 & \textit{elsewhere} \end{cases}$$

(a) Determine the value of k. [4]

(b) Determine the probability that a person will take longer than 115 minutes to reach the next town. [2]